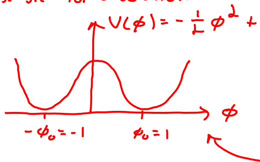
Tuesday, March 08, 2016
6:18 AM

"Broken" Symmetry

It is possible for a solution to the equations of motion to <u>not</u> exhibit a symmetry of the original action.

Example:



$$\mathcal{L} = \partial_\mu\phi\partial^\mu\phi - V(\phi) \Rightarrow \frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right) = -\frac{\partial V}{\partial\phi} - \frac{1}{2}\partial_\mu\partial^\mu\phi = 0$$

symmetric under $\phi \to -\phi$

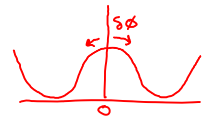
If we look for static solutions, i.e. $\partial_\mu\phi = 0$ then we want $\frac{\partial V}{\partial\phi} = -\phi + \phi^3 = 0 \Rightarrow \phi = \begin{cases} 0 & \text{symmetric} \\ +1 & \text{not symmetric} \\ -1 & \text{not symmetric} \end{cases}$

This is even more explicit when we focus on small fluctuations (as we do when we study particle-like behavior of the underlying fields in the SM).

Consider $\phi(x) = \phi_0 + \delta\phi(x)$ where ϕ_0 is one of the static solutions above. To determine the equation of motion for the fluctuations $\delta\phi(x)$, we simply stuff this back into the original Lagrangian.

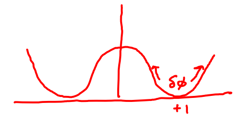
$$\mathcal{L}(\delta\phi) = \partial_\mu(\phi_0 + \delta\phi)\partial^\mu(\phi_0 + \delta\phi) - (\phi_0 + \delta\phi)^2 + (\phi_0 + \delta\phi)^4$$

$$= \cancelto{0}{\partial_\mu\phi_0}\partial^\mu\phi_0 + \cancelto{0}{\partial_\mu\phi_0}\partial^\mu\delta\phi + \partial_\mu\delta\phi\cancelto{0}{\partial^\mu\phi_0} + \partial_\mu\delta\phi\partial^\mu\delta\phi - (\phi_0 + \delta\phi)^2 + (\phi_0 + \delta\phi)^4$$



$$\mathcal{L}_0 = \partial_\mu\delta\phi\partial_\mu\delta\phi - \delta\phi^2 + \delta\phi^4$$

Same as original $\mathcal{L}$ for ϕ $\Rightarrow$ $\underbrace{\mathcal{L}(\delta\phi) = \mathcal{L}(-\delta\phi)}_{\text{still symmetric}}$



$$\mathcal{L}_{+1} = \partial_\mu\delta\phi\partial_\mu\delta\phi - (1 + \delta\phi)^2 + (1 + \delta\phi)^4$$

$\mathcal{L}$ for $\delta\phi$ has symmetry broken $\Rightarrow \mathcal{L}(\delta\phi) \neq \mathcal{L}(-\delta\phi)$

It is imperative to realize that the <u>full</u> underlying potential in <u>both</u> cases is symmetric. It's just then when we focus on <u>fluctuations about a particular solution</u> that the symmetry is not manifest, i.e. it appears to be broken.

Recall that for the complex scalar Higgs we had:

$$\mathcal{L}(\phi,\phi^*,A_\mu) = \tfrac{1}{2}\left[(\partial_\mu - \tfrac{ig}{\hbar c}A_\mu)\phi^*\right]\left[(\partial^\mu + \tfrac{ig}{\hbar c}A^\mu)\phi\right] - \tfrac{1}{2}\mu^2\phi^*\phi + \tfrac{1}{4}\lambda^2(\phi^*\phi)^2 + \tfrac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$

This has a symmetric solution: $\underbrace{\phi=0}_{\phi_1=0,\ \phi_2=0}$, $A_\mu=0$ where the $\mathcal{L}(\delta\phi,\delta\phi^*,\delta A_\mu)$ looks just like the expression above.

We also considered the more interesting solution: $\phi_1=\frac{\mu}{\lambda}$, $\phi_2=0$, $A_\mu=0$ where the Lagrangian for fluctuations $\delta\phi_1=\eta$, $\delta\phi_2=\beta$, $\delta A_\mu = A_\mu$ become:

$$\mathcal{L}(\eta,\beta,A_\mu) = \underbrace{\tfrac{1}{2}\partial_\mu\eta\partial^\mu\eta + \mu^2\eta^2}_{\text{massive scalar}} + \underbrace{\tfrac{1}{2}\partial_\mu\beta\partial^\mu\beta}_{\text{massless scalar}} + \underbrace{\tfrac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}\left(\tfrac{g}{\hbar c}\tfrac{\mu}{\lambda}\right)^2 A_\mu A^\mu}_{\text{massive vector}} + \text{various interactions}$$

What we have done here is pretty much like what we just discussed for symmetry breaking with the difference that here we are breaking a continuous symmetry, i.e. $U(1)$, where before the symmetry was discrete, $\phi \to -\phi$.

A picture will help...

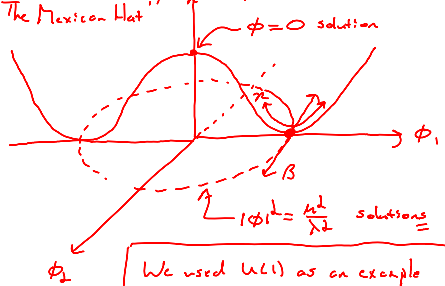
Monday, April 23, 2012
7:35 PM

The Higgs Potential

wrong sign mass term is now interpreted as an instability!

"The Mexican Hat"

$$\mathcal{U}(\phi) = -\tfrac{1}{2} m^2 \phi^* \phi + \tfrac{1}{4} \lambda^2 (\phi^* \phi)^2$$



$\boxed{\text{We used } U(1) \text{ as an example since it makes visualization easier.}}$

The mass of a fluctuation can be associated w/ the coefficient of the quadratic term.

$$\mathcal{U}(\phi) = A + B(\phi - \phi_0) + m^2(\phi-\phi_0)^2 + D(\phi-\phi_0)^3 + \dots \Rightarrow \left.\frac{d^2 \mathcal{U}}{d\phi^2}\right|_{\phi=\phi_0} = 2m^2$$

But the 2nd derivative is just expressing concavity!

Note: 2nd derivative >0 for η
$=0$ for β

What happened to original $\phi \to e^{i\theta(x)}\phi$?

The original symmetry is now encoded by "shifts" in β, i.e. $\beta \to \beta + \delta\beta$.

But this is a nonphysical gauge symmetry, so we can set $\beta = 0$ leaving only η, A_μ where $m_A \neq 0$!

In words: This "Higgs Mechanism" gives mass to the gauge fields of a "spontaneously broken" gauge symmetry through the coupling to an extra Higgs field ϕ (which of course has its own particle!).

The 2-polarization massless spin-1 gauge field "eats" the spin-0 Goldstone boson to get its 3rd polarization d.o.f. which is required when $m_A \neq 0$.

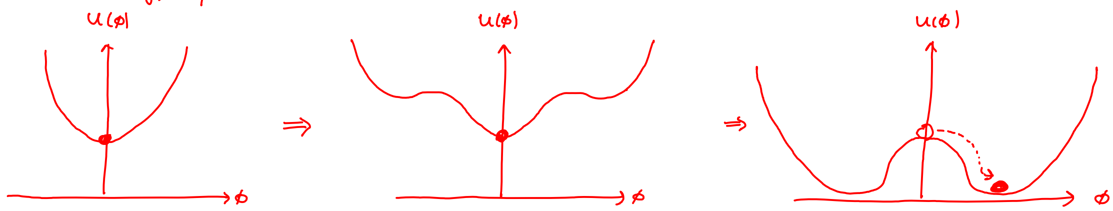
This simple $U(1)$ example can be generalized to the breaking of $SU(2)_L \times U(1)_Y \to U(1)_{EM}$ explaining the masses of the $W^{\pm}, Z^0$ bosons.

Furthermore, by coupling the fundamental matter fermions (taken to be massless for $SU(2)_L$ invariance) we can generate effective masses for these as well.

This still leaves the question: "How did the Higgs field ever get into the unstable solution in the first place?"

Wednesday, March 23, 2016
7:14 AM

Suppose that the Higgs potential itself has "evolved" over the history of the universe.



Lowest energy solution is symmetric
$SU(2)_L \times U(1)_Y$

Still symmetric
$SU(2)_L \times U(1)_Y$

Lowest energy solution(s) have broken symmetry
$U(1)_{EM}$

$\longrightarrow$ time

How does this happen? Recall that as the universe ages, it expands <u>and</u> cools, hence the average energy density is decreasing.

So we can consider: $\longrightarrow$ Energy

One of the important things we will discover when we start doing calculations is that the "constant" coefficients in our Lagrangian actually change with the energy scale. But constants like μ and λ are what determine the shape of the Higgs potential!

This might all sound strange, but you are probably already familiar with an example of this...

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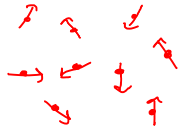
7:29 AM

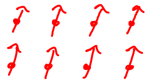
Consider a solid of magnetic dipoles at very high temperature. In this case the thermal motion is so extreme, that it overcomes the dipole-dipole interaction and everything looks random.

High T :



As we cool this system, the thermal motion eventually slows and is overtaken by the dipoles tendency to align.

Low T :



What is perhaps counter-intuitive is that the high T state is actually more symmetric! It has SO(3) invariance.
At low T there is a preferred axis in space leaving only SO(2) invariance.
So SO(3) is spontaneously broken to SO(2) in this system.

All of this can be modelled in terms of an "effective potential" for the dipole alignment in perfect analogy to the Higgs.

Note: The final preferred axis is completely undetermined in this case, hence the name spontaneous symmetry breaking.